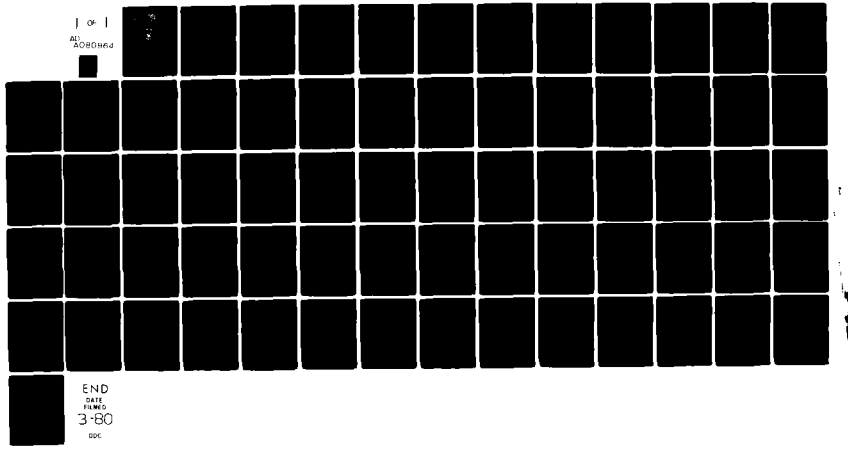
AD-A080 864 CALIFORNIA UNIV CA DEPT OF BUSINESS ADMINISTRATION F/G 5/9
A STEADY STATE LONGITUDINAL MANPOWER PLANNING MODEL WITH SEVERA--ETC(U)
AUG 79 R C GRINOLD N62271-79-M-1981

UNCLASSIFIED NPS-55-79-032 NL



1 OF 1
AD A080864

END
DATE FILMED
3-80
DDC

ADA080864

DDC FILE COPY

(18) (19) NPS55-79-025

(2)

LEVEL II

NAVAL POSTGRADUATE SCHOOL

Monterey, California

DDC
FEB 20 1980
E

(15) N62-71-0-M-1

(9) Technical rpt.

(6) A STEADY STATE LONGITUDINAL MANPOWER PLANNING MODEL WITH SEVERAL CLASSES BY MANPOWER.

by

(10) Richard C. Grinold

(11) Aug 79

(12) 69

Approved for public release; distribution unlimited.

80 2 19 111

411572

NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA

Rear Admiral T. F. Dedman
Superintendent

J. R. Borsting
Provost

This work was funded under Contract No. N62271-79-M-1981 in support of the Manpower Research Program of the Naval Postgraduate School. This program is funded by the Office of the Deputy Assistant Secretary of the Navy and by the Office of the Deputy Chief of Naval Operations (OP-01).

Reproduction of all or part of this report is authorized.

This report was prepared by

Richard Grinold

Richard Grinold, Professor
Finance & Management Science,
Graduate School of Business
University of California, Berkeley

Reviewed by:

Michael G. Sovereign

Michael G. Sovereign, Chairman
Department of Operations Research

Released by:

William M. Tolles

William M. Tolles
Dean of Research

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NPS55-79-025	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A Steady State Longitudinal Manpower Planning Model with Several Classes by Manpower		5. TYPE OF REPORT & PERIOD COVERED Technical 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Richard C. Grinold (Calif (Univ, Berkeley. Dept of Bus. Admin (See p. 1))		8. CONTRACT OR GRANT NUMBER(s) N6227l-74-M-1981
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		12. REPORT DATE August 1979 13. NUMBER OF PAGES 67
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Manpower Planning
Linear Programming
Goal Programming
Quadratic Optimization
Longitudinal Models

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In the Navy's officer system there are several classes of manpower (e.g. pilots and submariners) that perform specialized jobs and also can perform several types of non-specialized jobs (e.g. military training, personnel management, etc.). This report is concerned with the general problem of allocating the different types of jobs among the several classes of manpower.

The report describes a model that constructs a personnel inventory by rank

OVER

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

mt 411592

20. Abstract continued

for each of several manpower classes (pilots, etc.) and then allocates those people to the specialized and common jobs that they are allowed to do.

The idea of the model is to allow a policy maker to quickly reconcile billet requirements with the reality of available accessions, job sharing targets between classes, and continuation rates of the different manpower classes.

Different allocations can be produced either by assuming the values of a few key variables, or they can be generated using an optimization scheme that sets "allowable" percentage errors. Four optimization variants based on this idea are described. The report contains some typical data, the results of calculations, and a description of computer programs used to solve the problem.

Accession For
NTIS GRA&I
DDC TAB
Unannounced
Justification
By
Distribution/
Availability
Dist
A

A STEADY STATE LONGITUDINAL MANPOWER PLANNING MODEL WITH SEVERAL CLASSES OF MANPOWER

by

Richard C. Grinold

August 1979

TABLE OF CONTENTS

		Page
1.	Introduction	1
2.	Manpower	6
3.	Stages	7
4.	Manpower Flows	8
5.	Jobs	10
6.	Billet Requirements	12
7.	Billet Sharing Array	13
8.	Target Allocations	14
9.	Actual Allocation	15
10.	Conservation	16
11.	Excess and Deficit	17
12.	People Sharing	18
13.	Stage Substitution	20
14.	Cost	21
15.	Review	22
16.	Choice of an Allocation	24
17.	Quality of an Allocation	25
18.	Piecewise Convex Penalties	28
19.	Quadratic Penalty Functions	30
20.	Cost Quality Trade-Off	32

TABLE OF CONTENTS Cont.

		Page
APPENDIX A:	LINEAR PROGRAMMING MODELS	34
APPENDIX B:	UNCONSTRAINED QUADRATIC MODELS	43
APPENDIX C:	SAMPLE SOLUTION	46
APPENDIX D:	ORGANIZATION OF PROGRAMS	52
REFERENCES	.....	58

A STEADY STATE LONGITUDINAL MANPOWER PLANNING MODEL WITH SEVERAL CLASSES BY MANPOWER

by

Richard C. Grinold

ABSTRACT

In the Navy's officer system there are several classes of manpower (e.g. pilots and submariners) that perform specialized jobs and also can perform several types of non-specialized jobs (e.g. military training, personnel management, etc.). This report is concerned with the general problem of allocating the different types of jobs among the several classes of manpower.

The report describes a model that constructs a personnel inventory by rank for each of several manpower classes (pilots, etc.) and then allocates those people to the specialized and common jobs that they are allowed to do.

The idea of the model is to allow a policy maker to quickly reconcile billet requirements with the reality of available accessions, job sharing targets between classes, and continuation rates of the different manpower classes.

Different allocations can be produced either by assuming the values of a few key variables, or they can be generated using an optimization scheme that sets "allowable" percentage errors. Four optimization variants based on this idea are described. The report contains some typical data, the results of calculations, and a description of computer programs used to solve the problem.

A STEADY STATE LONGITUDINAL MANPOWER PLANNING MODEL WITH SEVERAL CLASSES OF MANPOWER

by

Richard C. Grinold

1. Introduction

This paper outlines the construction of a manpower planning model for a system in which each of several classes of manpower are assigned to several categories of jobs. The jobs are either specialized in that only one type manpower can perform the job or general in that more than one manpower type is qualified to do the job.

Both jobs and manpower are broken down by experience level. A job with a certain experience level must be filled with a person that has the same experience level.

The model assumes a steady state. In each year the job requirements are the same. In each year the same number of people enter the system (bottom level entry only), the same number leave the system from each experience level, and the same number move up from each one level so that the inventory of people at each exerpience level and in each manpower class remains the same.

There are several items represented in the model:

(i) The requirements for jobs by experience levels.

(ii) The requirements for people by experience levels.

(iii) The job sharing targets for common jobs; the fraction of common jobs we would like to see allocated to each manpower class.

(iv) The people sharing targets for manpower classes; the fraction of each manpower class that we would like to see allocated to each type of job.

(v) The rate of accessions in each manpower class.

(vi) The retention of manpower in each class; i.e. the relationship between accessions in each class and the inventory of people by experience level in each class.

The model is designed to study the interaction of these factors. It is easy to see from the list above that items (i) through (iv) are interrelated and unless care is taken in specifying these goals they will be inconsistent. Also items (ii), (v) and (vi) are strongly related in that (v) and (vi) determine the inventory of people in each class at each experience level. Only in the most fortunate circumstances will this be consistent with the personnel requirements (ii).

Items (i)-(iv) are targets. They may be unrealistic and inconsistent but they can be set by the manpower planner. Items (v) and (vi), however, depend to some extent on the behavior of personnel in the system (vi) and on the system's ability to attract qualified people to each class (v). The reader

should keep in mind this caveat and be prepared for shifts in personnel behavior in response to changes in other system parameters or in external factors.

The model is a laboratory for testing the relationship between these factors. We can try one set of policies and look at the discrepancy between job requirements and actual requirements as well as the discrepancy between personnel requirement and the actual inventory in each class.

The model may be used in several ways. The easiest is for the planner to stipulate a policy and then examine its effects. A more difficult procedure is for the planner to stipulate a range of policies and then use some type of optimization scheme to select a policy within the range. This paper contains four examples of this type of optimization. Each is based on the notion of a penalty function the measures the discrepancy between desired job and personnel requirements and what is actually provided. We take two types of penalty functions: piecewise linear and quadratic, and we examine two broad sets of policies: allocating people to jobs using the job sharing rules ((iii) above), or allocating people to jobs using the people sharing rates ((iv) above). This gives us four combinations and thus four distinct optimization models. The piecewise linear penalty functions lead to linear programming models and the quadratic penalty functions to the minimization of a quadratic form subject to linear equality constraints.

The model is motivated by a study of the U. S. Navy's officer corps. We shall carry an example using that system throughout the text in order to illustrate each idea.

The model is based on a longitudinal manpower flow model. This type of system is described in depth in Grinold and Marshall [1], Chapter 3. The use of piecewise linear penalty functions to measure the discrepancy between actual performance and stated objectives is commonly called "goal programming." This idea has been extensively developed by Charnes and Cooper; a good review can be found in [3]. The use of quadratic penalty functions is quite common in the optimal control literature; its use in more behavioral settings was pioneered by Holt, Modigliani, Muth and Simon in [2].

The paper consists of several short sections each dedicated to a specific point and most illustrated by the example carried thoughout the text. The model's structure is given in Sections 1-14. Section 15 is a review that gathers all the definitions presented to that point. Sections 16 to 20 describe alternative ways to choose an allocation, the idea behind our use of penalty functions, and the specific construction of the piecewise linear and quadratic penalty functions.

Appendix A describes the two linear programming models that arise from the use of piecewise linear penalties and Appendix B the two models that stem from the use of quadratic penalties. Appendix C contains some sample solutions.

The organization reflects the relative importance of the topics. The structure of the model is the most important and it is stressed and reviewed in Sections 2-14. The use of optimization to select an allocation is, in the main, merely a device to circumvent the difficulty of having a wide range of policy choice. Once the optimization rule is set the planner has a direct route from policy to result. It is the variation in results that comes from changes in policy that will be of most interest to the planner. The optimization is a device to help make that connection.

One final point should be made before describing the model's structure. This is an aggregate planning model. It is intended to test policies that will, in turn, provide a foundation for the day-to-day operating of the system. This model certainly will not show us how to operate the system. No model can answer all questions simultaneously and ours is no exception.

2. Manpower

There are K different classes of manpower indexed by $k = 1,2,\ldots,K$. The classification scheme is, of course, directly related to the objectives of the model builder. In general, the classification should be fine enough to capture the important substitution possibilities and economical in avoiding the listing of all possibilities in a futile attempt to replicate reality. In our example there are five manpower types.

Manpower Type	Officer Designator	Description
1	110x	Women
2	111x (and 116x)	Surface warfare
3	112x (and 117x)	Submarine warfare
4	131x and 139x	Pilots
5	132x and 137x	Naval flight officers

Table 2-1: The manpower classes.

3. Stages

Each officer's career is broken down into I-stages indexed by $i = 1,2,\ldots, I$. The stages can be defined in many ways. The simplest is by period of service; if we track officers for 26 years then the index i would go from one to 26. In our example the stages roughly coincide with the time period in which officers hold a certain rank. In general for $i = 1,2,\ldots, I$, stage i will run from time of service s_{i-1} to s_i. The maximum length of service is $s_I = S$ and $s_0 = 0$.

A person in manpower class k and in stage i of their career will be called a type (i,k) person.

Stage	LOS	Description
1	0-2	Ensign
2	2-4	Lieutenant--J.G.
3	4-9	Lieutenant
4	9-14	Lieutenant Commander
5	14-19	Commander
6	19-26	Captain

Table 3-1. Stages or experience levels.

4. Manpower Flows

We assume a steady state model. In each year the same number of people will enter each manpower class, and the inventory of people in each year of service and in each manpower class will remain constant.

For class $k = 1,2,\ldots,K$ let y_k be the number of people entering manpower class k, and let $\alpha(k,s)$ be the fraction of these accessions that remain in the system for s years. The index s runs from 0 to S years and the function $\alpha(k,s)$ is decreasing in s. The function $\alpha(k,s)$ is frequently called the survivor curve.

Stage i of a person's career runs from length of service s_{i-1} to s_i ($s_0 = 0$). The number of people in stage k is $w_{ik}y_k$ where w_{ik} is defined by

$$w_{ik} = \int_{s_{i-1}}^{s_i} \alpha(k,s)ds \ .$$

In our example, let's take a hypothetical survivor curve for pilots (class 4). We follow these officers for 26 years. Figure 4-1 shows the survivor curve. The shaded areas are integrals under this curve for the duration of each stage.

The calculation of w_{ik} can be approximated using a discrete form of the survivor curve.

The coefficients w_{ik} can be interpreted as the amount of time a person in manpower class k is expected to spend in stage i. A crude way to view this is to say that w_{ik} is the

product of the probability of reaching stage i, and the length of stage i. The probability will be $w_{ik}/(s_i - s_{i-1})$.

	i	w_{ik}	$s_i - s_{i-1}$	Prob. of reaching stage i
S	1	1.97	2	0.985
T	2	1.85	2	0.925
A	3	2.50	5	0.5
G	4	1.40	5	0.28
E	5	1.22	5	0.25
S	6	0.54	7	0.77

Table 4-1. The expected waiting time.

5. Jobs

There are J different types of jobs indexed by $j = 1,2,\ldots,J$. In our example we consider seven job types:

Job No.	Billet Code	Description
1	1000	General, nonwarfare billets
2	1050	General, warfare billets
3	1110 and 1160	Surface
4	1120 and 1170	Subsurface billets
5	1310 and 1390	Pilots
6	1320 and 1370	Naval Flying Officer
7	1300	General Aviation

Table 5-1. Definition of Jobs

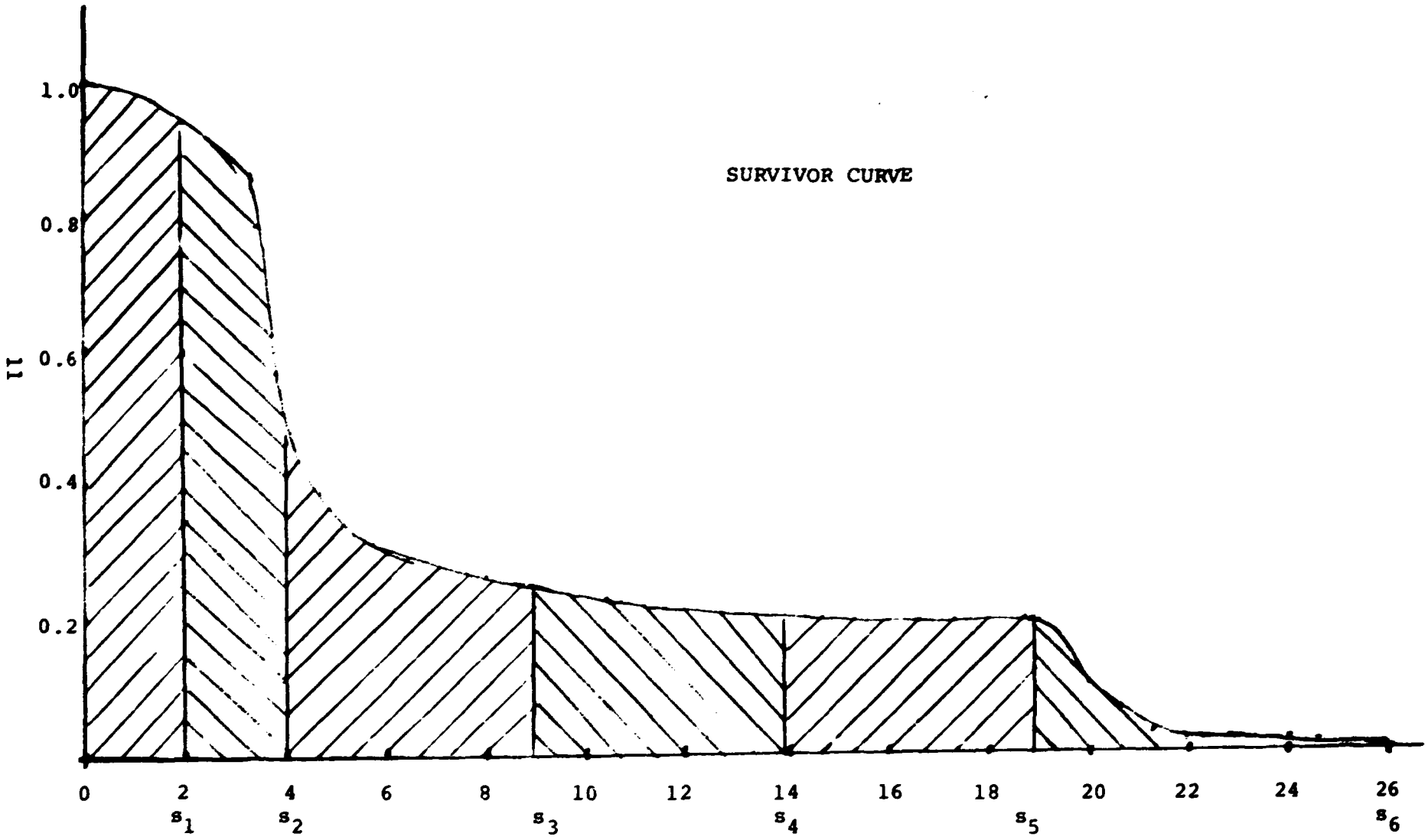


Figure 4-1. Length of service

6. Billet Requirements

The I by J matrix B contains the billet requirements. Since the career stage is a measure of experience (e.g. length of service or rank), the requirements are broken down by stage.

b_{ij} the number of stage i people need for jobs type j

The billet requirements matrix for our example is shown below.

The billets will be identified by the index pair (i,j). Thus we shall speak of type (i,j) billets and type (i,k) people.

		JOBS						
		1	2	3	4	5	6	7
STAGE	1	409	0	2095	688	961	521	0
	2	1008	0	1883	734	1998	1212	1
	3	1806	378	2080	844	3572	1399	636
	4	1495	599	1464	810	1780	547	726
	5	1031	470	926	501	678	55	902
	6	592	490	321	149	0	0	387

Table 6. The billet requirements matrix B.

7. Billet Sharing Array

For each i the K by J matrix F^i gives the share (fraction) of the type (i,j) billets that should be performed by manpower type k. The elements of F^i are denoted f^i_{kj}, they are nonnegative, and the rows sum to one, $\sum_k f^i_{kj} = 1$.

If $f^i_{kj} > 0$, we interpret the fraction as a goal; we would like that fraction of the (i,j) billets filled from manpower class k. However, we interpret $f^i_{kj} = 0$ to mean that manpower class k is not qualified to fill billet (i,j).

There is also possible confusion between the idea of job sharing and a people sharing concept. A manpower class may perform two types of jobs: the jobs for which it is uniquely qualified and other jobs that are shared among several manpower classes. Another way to look at the allocation of the common jobs is to stipulate a fraction of the inventory of (i,k) people that should be assigned to job j. That idea will be considered in Section 12.

To save space, we shall only give one of the matrices F^i (for $i = 3$) used in our example.

JOBS

MANPOWER CLASS		1	2	3	4	5	6	7
	1	0.7						
	2	0.15	.5	1				
	3	0.06	.2		1			
	4	0.05	.18			1		.57
	5	0.04	.12				1	.43

Table 7-1. The job sharing matrix F^i for $i = 3$.

8. Target Allocations

The number of (i,k) people desired in job (i,j) is given by

$$t^i_{kj} = f^i_{kj} b_{ij} .$$

Recall that b_{ij} is the (i,j) billet requirement and f^i_{kj} is the fraction of those billets to be filled by manpower class k.

The target allocations used in our example (for i = 3) are shown in Table 8-1.

JOBS

MANPOWER CLASS	1	2	3	4	5	6
1	1264.2					
2	270.9	189	2080			
3	108.36	75.6		844		
4	90.3	68.04			3572		362.52
5	72.24	45.36				1399	273.48

Table 8-1. The target allocation t^3_{kj}

9. Actual Allocation

Let the variable a^i_{kj} give the actual number of (i,k) people assigned to billet (i,j). If it was possible we would like $a^i_{kj} = t^i_{kj}$. That would ensure that all targets are met. That, however, is usually impossible. Indeed, reconciling the a^i_{kj} and t^i_{kj} is the purpose of this model.

The variables a^i_{kj} do not mean that a^i_{kj} individuals from class k are locked into type (i,j) billets when they are in stage i. It does mean that at any time a^i_{kj} people from class k are filling type (i,j) billets. Any particular (i,k) person, may spend stage i in several billets.

If $f^i_{kj} = 0$, then an (i,k) person is not qualified to fill an (i,j) billet. So if $f^i_{kj} = 0$, we shall require that $a^i_{kj} = 0$.

10. Conservation

Recall that $w_{ik}y_k$ is the number of type (i,k) people. The allocation must satisfy the following conservation relation

$$\sum_{j=1}^{J} a_{kj}^{i} = w_{ik}y_k \qquad \text{for all} \quad (i,k)$$

This says that all manpower fills some job.

11. Excess and Deficit

One of our main objectives in constructing this model is to compare the allocation and target. We could look at all the discrepancies $a^i_{jk} - t^i_{jk}$. That would be very general, but it may not be comprehensible. We have chosen to concentrate on more aggregate measures of excess and deficit: the actual number of (i,k) people and the actual number of (i,j) billets filled as compared with the targets. Our notation for these measures is

$x_{ij} = \sum_{k=1}^{K} a^i_{kj}$	the number of poeple assigned to billet (i,j)
$x_{ij} - b_{ij}$	the discrepancy between billet (i,j) assignment and requirements
$z_{ik} = \sum_{j=1}^{J} a^i_{kj}$	the number of (i,k) people
$p_{ik} = \sum_{j=1}^{J} t^i_{kj}$	the total requirement for type (i,k) people
$z_{ik} - p_{ik}$	the discrepancy between the inventory of (i,k) people and requirements.

12. People Sharing

Recall that we defined our targets from the viewpoint of billet sharing. Given any allocation we can calculate the people sharing fractions. Let g^i_{kj} be the fraction of type (i,k) people that are assigned to job j, thus billet (i,j). The g^i_{kj} are given implicitly by the equation

$$a^i_{kj} = g^i_{kj} z_{ik}$$

As mentioned above, an alternative model could be constructed using the people sharing fractions g^i_{kj} as input targets and the billet sharing fractions f^i_{kj} as outputs. In that case we would start with people requirement p_{ik}, and the people sharing rules g^i_{kj} and define target allocations by

$$t^i_{kj} = g^i_{kj} p_{ik}.$$

Then we can calculate b_{ij} from

$$b_{ij} = \sum_k g^i_{kj} p_{ik}.$$

In any case, for consistency, the following relations should hold

(i) $g^i_{kj} p_{ik} = f^i_{kj} b_{ij}$

(ii) $\sum_{j=1}^{J} g^i_{kj} = 1 \Longrightarrow p_{ik} = \sum_{j=1}^{J} f^i_{kj} b_{ij}$

(iii) $\sum_{k=1}^{K} f^i_{kj} = 1 \Longrightarrow b_{ij} = \sum_{k=1}^{K} g^i_{kj} p_{ik}$.

To assure consistency we should either start with f and b and then calculate g and p, or start with g and p and calculate f and b. If we have a desire to start with g and b, then one suggestion is to guess p, see if it works, and then do some revision.

From the values of b and f (for all i) used in this paper, we can compute the personnel requirements, shown below.

		Manpower Class				
		1	2	3	4	5
S	1	286	2156	713	981	537
T	2	706	2034	794	2049	1253
A	3	1264	2540	1028	4093	1790
G	4	1046	1988	1019	2376	991
E	5	722	1316	657	1328	540
	6	414	655	283	338	249

Table 12-1. The personnel inventories, p_{ik}.

13. Stage Substitution

The astute reader will notice that we have not allowed substitution between stages. Thus a type (i,k) person is not allowed to fill a type (i+1, j) billet or a type (i-1, j) billet.

Our formuation actually allows such assignments, however, we are assuming that they net out to zero. That is for each (i,k) person filling a stage i+1 billet, there is an (i+1,k) person filling a stage i billet.

One way to study the possibility of stage substitution is to vary the length of the intervals (s_{i-1}, s_i) that define the stages. However, the billet requirement are presumably set with some experience level in mind. Therefore major shifts in the definitions of stages would require adjustments in the billet requirements.

14. Cost

The cost of any allocation can be calculated using

$$\sum_i \sum_k c_{ik} w_{ik} y_k ,$$

where c_{ik} is the annual cost of an (i,k) person and $w_{ik} y_k = z_{ik}$ is the number of (i,k) people. The cost data c_{ik} should include salary, benefits, retirement; training, promotion, and recruitment costs. Notice we can define c_k as $\sum_k c_{ik}\ w_{ik}$, and then rewrite the cost as $\sum_k c_k y_k$. We interpret c_k as the average career cost of each accession into manpower class k.

15. Review

We have constructed our model. The remainder of the paper is devoted to ways of calculating particular assignments and contains a sample calculation.

We summarize here the notations and definitions presented to this point.

manpower class	indexed by $k = 1,2,\ldots,K$
stages of a career	indexed by $i = 1,2,\ldots,I$ defined by length of service $(s_1, s_2, \ldots, s_I)$
jobs	indexed by $j = 1,2,\ldots,J$
people	indexed by stage and class (i,k)
accessions	y_k, the number of people entering class k per year
stage inventory	$w_{ik}y_k$ is the number of (i,k) people; y_k is the accession rate and w_{ik} is obtained from the survivor curve and the stage definition. Note $z_{ik} = w_{ik}y_k$.
billets requirements	indexed by stage and job (i,j); b_{ij} is the billet requirements
billet sharing	indexed by (i,j,k); f^i_{kj} is the fraction of (i,j) billets that should be filled from class k.

target allocation	t^i_{kj}. The number of (i,k) people desired in billet (i,j)
actual allocation	a^i_{kj} the number of (i,k) people in billet (i,j)
class sums	x_{ij} and b_{ij} are respectively the number assigned to billet (i,j) and the requirement for billet (i,j)
job sums	z_{ik} and p_{ik} are respectively the number of (i,k) people, and the target for (i,k) people.
people sharing	g^i_{kj}, the fraction of (i,k) people assigned to billet (i,j); $a^i_{kj} = g^i_{kj} z_{ik}$.
cost	c_{ik} is the annual cost of an (i,k) person. The cost of a particular allocation is $\sum_i \sum_k c_{ik} z_{ik}$.

16. Choice of an Allocation

Up to this point we have set out a model structure. The next task is to devise one or two more procedures for selecting allocations. There are two general approaches to this task.

On approach is to use an ad hoc rule. For example, we could fix accessions at projected rates for each class. That would fix the variables y_k and thus $z_{ik} = w_{ik}y_k$. Next we could specify the people sharing rules q^i_{kj}. That would give us the allocation $a^i_{kj} = q^i_{kj}z_{ik}$.

The second approach is to use some type of optimization to select a policy. This optimization is either based on a trade-off between the cost of any allocation and some measure of its quality, or simply a measure of the quality of the allocation.

17. Quality of an Allocation

In order to choose an allocation using optimization we need some measure of the quality of that allocation.

In our model we have already decided to focus on the discrepancy between people inventory and targets $(z_{ik} - p_{ik})$ and the discrepancy between billet assignments and requirements $(x_{ij} - b_{ij})$.

We have selected the simplest form of quality measure that makes sense. It is a penalty function that is zero for a perfect allocation and positive for others. Thus it measures the lack of quality. The penalty function has three properties.

(1) The penalty can be written as

$$\sum_i \sum_k h_{ik}(z_{ik}) + \sum_i \sum_j \ell_{ij}(x_{ij})$$

where

(2) $h_{ik}(z_{ik})$ and $\ell_{ij}(x_{ij})$ are nonnegative, convex and equal to zero if $z_{ik} = p_{ik}$ and $x_{ij} = b_{ij}$ respectively.

(3) Parameters θ_{ij} and ψ_{ij} measure a unit upper and lower percentage error in meeting billet requirements

$$\ell_{ij}(x_{ij}) = 1 \quad \text{if} \quad x_{ij} = (1 + \theta_{ij})b_{ij}\ , \quad \text{or} \quad x_{ij} = (1 - \psi_{ij})b_{ij}\ .$$

Similar parameters ϕ_{ik} and δ_{ik} are used for discrepancies between actual inventories of people and requirements.

$$h_{ik}(z_{ik}) = 1 \quad \text{if} \quad z_{ik} = (1 + \phi_{ik})p_{ik} \ , \quad \text{or} \quad z_{ik} = (1 - \delta_{ik})p_{ik} \ .$$

Item (1) above says that the quality measure is separable, item (2) that is nonnegative, convex, and zero if the assignment is exactly on target. The third item requires more discussion. Item (3) is designed to answer the question: How do we compare a 4 percent shortfall in meeting a critical target with a 10 percent short fall in meeting a less critical target? Our answer to this question is to take a single target as a benchmark and to say arbitrarily for that target that a certain percent over and a certain percent under the target yields an error of one. Then for any other target we can compare the seriousness of deviations with our benchmark. We say a fraction θ_{ij} over target or ψ_{ij} under target is as serious as the deviations we have established for our benchmark.

For example, we could take stage 3 pilots as our benchmark ($i = 3$, $j = 5$), and take the percentage under as 4% ($\psi_{35} = 0.04$) and percentage over as 10% ($\theta_{35} = 0.10$) as the unit serious deviation for our benchmark. Now take any other category, for example, stage 4 general warfare billets ($i = 4$, $j = 2$). Then we ask how much under target would the assignment to these billets have to be in order to be as

serious as a 4% shortage of stage 3 pilots. In this way we can try to make the essential judgments about the trade-offs between different categories.

We shall now give two examples of penalty functions and then give some practical examples of how such a criterion could be used.

18. Piecewise Convex Penalties

One of the simplest ways to construct the penalty measure for deviations from target is to use a piecewise convex function. To simplify notation we should consider the case of a discrepancy between the inventory of people and desired inventories and we shall drop the (i,k) subscripts.

We need a piecewise linear convex function $h(z)$ that satisfies

(i) $h(z) = 0$ if $x = p$

(ii) $h(z) = 1$ if $x = (1 + \phi)p$

(iii) $h(z) = 1$ if $x = (1 - \delta)p$

Such a function is shown in Figure 2. This function can be represented in several ways. One of the simplest is as the maximum of two linear functions

$$h(z) = \max \left[\frac{1}{\phi} \left(\frac{z-p}{p}\right) , \frac{1}{\delta} \left(\frac{p-z}{p}\right) \right]$$

This functional form is flexible and easy to manipulate. We shall see below, that it can be used to obtain an allocation using a linear program. Two examples of this are contained in the appendix.

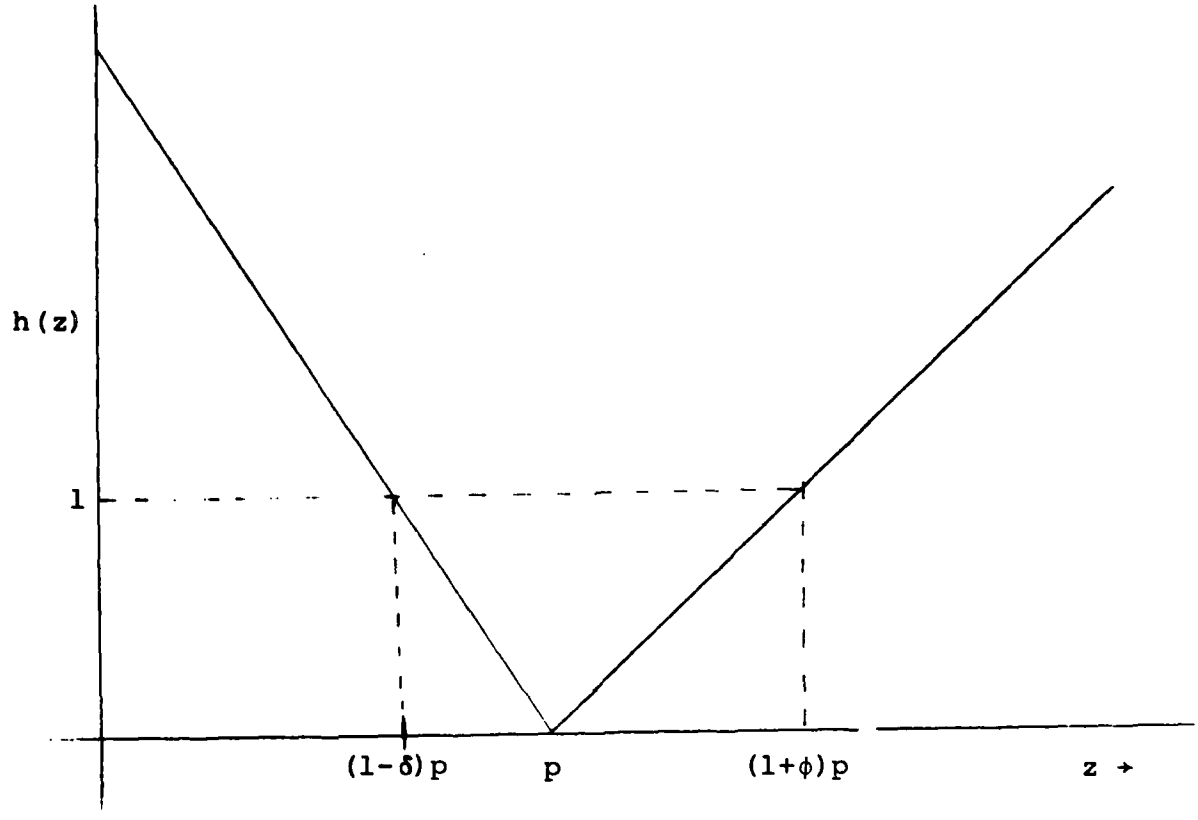


Figure 18-1. Piecewise linear convex function.

19. Quadratic Penalty Functions

Quadratic penalty functions are also quite easy to use. Unfortunately, we cannot get a quadratic function to satisfy all of our conditions. For example let us write

$$\ell(x) = \alpha(x - b)^2 + \beta(x - b) + \gamma.$$

We want to have

(i) $\ell(x) = 0$ if $x = b$

(ii) $\ell(x) = 1$ if $x = (1 + \theta)b$

(iii) $\ell(x) = 1$ if $x = (1 - \psi)b$

(iv) $\ell(x) \geq 0$

where item (iv) implies $\ell(x)$ has its minimum at $x = b$.

These four conditions cannot be met by a quadratic function unless $\theta = \psi$; that is unless there is a symmetry between being under and over target. In the symmetric case we can write $\ell(x)$ as

$$\ell(x) = \left(\frac{x - b}{\theta b}\right)^2 \qquad \text{when} \quad \theta = \psi.$$

When the penalties are not symmetric, we must relax one of our four conditions. If we relax condition (iv) then $\ell(x)$ becomes

$$h(x) = \frac{1}{\theta\psi}\left(\frac{x - b}{b}\right)^2 - \frac{1}{2}\frac{(\theta - \psi)}{\theta\psi}\left(\frac{x - b}{b}\right) .$$

This function has its minimum midway between $(1 + \theta)b$ and $(1 - \psi)b$ and it has a value of

$$-\frac{(\theta - \psi)^2}{2\theta\psi}$$

at the minimum.

This approximation is useful if θ and ψ are similar. However, the approximation becomes much worse when θ and ψ are much different.

Two quadratic penalty models are described in the appendix.

20. Cost Quality Trade-Off

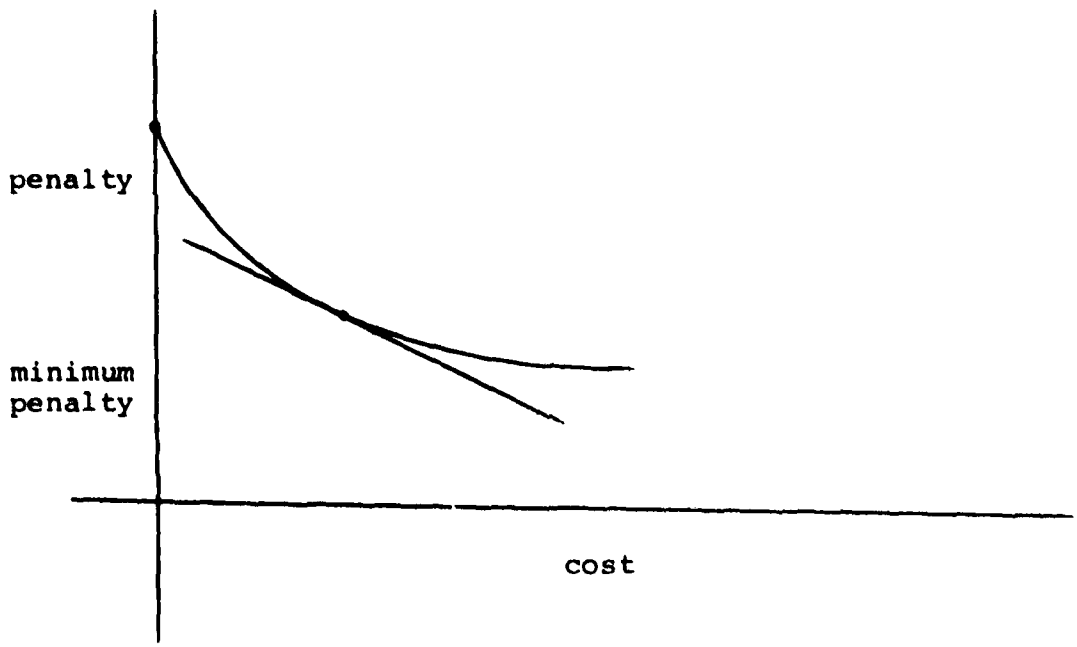
We saw above that any particular allocation has an annual cost and also a penalty associated with its deviation from targets for billet requirements and personnel inventories. By placing different weights on the cost and penalty we can obtain a family of objectives that will lead to efficient allocations.

The cost-penalty frontier is shown below. Notice the minimum cost solution is zero, since we would not have any accessions; i.e. $y = 0$, thus no cost.

We write our objective as

$$\lambda \sum_k c_k y_k + (1-\lambda) \{ \sum_i \sum_k h_{ik}(z_{ik}) + \sum_i \sum_j \ell_{ij}(x_{ij}) \} ,$$

As the parameter λ increases from 0 to 1 more emphasis is placed on the cost and less on the penalty. At $\lambda = 0$, we minimize the penalty and ignore the cost; at $\lambda = 1$ we minimize cost and ignore the penalty. For intermediate values of λ we establish a trade-off between cost and penalty. This curve should be used with some caution since the costs are expressed in real units (dollars per year), but the penalty is expressed in ad-hoc units.


penalty
minimum
penalty
cost

APPENDIX A

LINEAR PROGRAMMING MODELS

This appendix describes two linear programming approaches for selecting an allocation. The discussion is brief and is intended for those familiar with linear programming. The first model is called LPB for billet sharing and the second is called LPP for people sharing.

In both models the idea is to choose the number of people assigned to billet (i,j), and the number of accessions in manpower class k; these are respectively denoted x_{ij} and y_k. The allocation a^i_{kj} is then fixed using an ad hoc rule. We have selected two rules and thus get two different linear programming models.

The first rule is for sharing billets

Rule B (billet share)

$$a^i_{kj} = f^i_{kj} x_{ij} \tag{A-1}$$

Recall that f^i_{kj} is the fraction of type (i,j) billets that we desire to have satisfied from manpower class k. Rule B allocates the error in meeting billet requirements among manpower classes so that there is a constant percentage error in each class's allocation. If there is a percentage error of η_{ij} in meeting the billet (i,j) requirement, i.e.

$$\eta_{ij} = \left(\frac{x_{ij} - b_{ij}}{b_{ij}} \right),$$

then the billet sharing rule means that the percentage error in the assignment of type k manpower to billet (i,j) will also be η_{ij}.

The people sharing rule is

Rule P

(A-2) $$a^i_{kj} = g^i_{kj} z_{ik}$$

where $z_{ik} = \alpha_{ik} y_k$ is the number of type (i,k) people. The type (i,k) people are allocated among different jobs using the people sharing parameters g^i_{kj}. If there is an error of η_{ik} in the inventory of type (i,k)

$$\eta_{ik} = \frac{(z_{ik} - p_{ik})}{p_{ik}}$$

then the percentage error in the assignment of type (i,k) people to job j will be η_{ik} (recall $t^i_{kj} = g^i_{kj} p_{ik}$). The billet sharing rule leads to a larger linear program than the people sharing rule.

The constraints of the linear program for billet sharing are given below. The objective is the same for both programs; it will be discussed after we examine the constraints.

Dual Variable	Constraint	Explanation
u_{ik}	$\sum_{j=1}^{J} f_{kj}^{i} x_{ij} - w_{ik} y_k = 0$ for all (i,k)	conservation of people
v_{ik}	$w_{ik} y_k + d_{ik} - e_{ik} = p_{ik}$ for all (i,k)	discrepancy between type (i,k) people inventory and requirement
q_{ij}	$x_{ij} + m_{ij} - n_{ij} = b_{ij}$ for all (i,j)	discrepancy between type (i,j) billet assignments and requirements
	$x_{ij}, m_{ij}, n_{ij}, d_{ik}, e_{ik}, y_k \geq 0$	

Table A-1. Constraints for LP-B.

The first set of constraints is simply the conservation of people. There will be $w_{ik}y_k$ type (i,k) people and under the billet sharing rule $f^i_{kj}x_{ij}$ of them will be assigned to billet (i,j).

The second set of constraints measures the difference between actual inventories of type (i,k) people $w_{ik}y_k$ and the requirement p_{ik}. The objective is selected so that the solution to the linear program will have e_{ik} positive and d_{ik} equal to zero if $w_{ik}y_k$ exceed p_{ik} and e_{ik} equal to zero and d_{ik} positive if $w_{ik}y_k$ is less than p_{ik}. The e_{ik} and d_{ik} thus measure the excess and deficit in the type (i,k) people account.

The variables m_{ij} and n_{ij} play a similar excess and deficit role in the type (i,j) billet account.

The first set of constraints for the LP-P program is

$$x_{ij} - \sum_{k=1}^{K} g^i_{kj} w_{ik} y_k = 0 \quad \text{for all } (i,j) . \tag{A-3}$$

This again is a conservation constraint. There are $w_{ik}y_k$ type (i,k) people and g^i_{kj} of them are assigned to job j. Thus these constraints assure that the assignments to billets, x_{ij}, are consistent with actual manpower available.

The second and third sets of constraints in LP-P are identical with the second and third sets of constraints in LP-B. They define the excess and deficit in people and billet accounts.

The form of the conservation constraint in LP-P allow for some simplifications. Notice that if all the y_k are nonnegative then (A-3) implies that all the x_{ij} will be nonnegative. Thus we can use (A-3) to eliminate x_{ij} from the problem. With this simplification the constraints of LP-P are shown in Table A-2.

The objective for both linear programs will typically be a compromise between minimizing costs and minimizing the penalty that measures our departure from people and billet targets. The cost objective is

$$\text{(A-4)} \qquad \sum_{k=1}^{K} c_k y_k \ .$$

The penalty measure is

$$\text{(A-5)} \qquad \sum_{i=1}^{I} \sum_{j=1}^{J} \pi_{ij} n_{ij} + \mu_{ij} m_{ij} + \sum_{i=1}^{I} \sum_{k=1}^{I} \gamma_{ik} e_{ik} + \sigma_{ik} d_{ik}$$

where

$$\text{(A-6)} \qquad \begin{aligned} &\text{(i)} \quad \pi_{ij} = (\theta_{ij} b_{ij})^{-1} \\ &\text{(ii)} \quad \mu_{ij} = (\psi_{ij} b_{ij})^{-1} \\ &\text{(iii)} \quad \gamma_{ik} = (\phi_{ik} p_{ik})^{-1} \\ &\text{(iv)} \quad \sigma_{ik} = (\delta_{ik} p_{ik})^{-1} \end{aligned}$$

Dual Variable	Constraint	Explanation
v_{ik}	$w_{ik}y_k + d_{ik} - e_{ik} = p_{ik}$ for all (i,k)	Type (i,k) people discrepancy
q_{ij}	$\sum_k g^i_{kj} w_{ik} y_k + m_{ij} - n_{ij} = b_{ij}$ for all (i,j)	Type (i,j) billet discrepancy
	$y_k, d_{ik}, e_{ik}, m_{ij}, n_{ij} \geq 0$	

Table A-2. Constraints for LP-P

Recall that θ, ψ, ϕ, and δ are the percentage errors that lead to a unit loss.

A balance between these two objectives is obtained by taking a weighted combination.

$$(A\text{-}7) \quad \lambda \sum_{k=1}^{K} c_k y_k + (1 - \lambda) \times \left\{ \sum_{i=1}^{I} \sum_{j=1}^{J} \pi_{ij} n_{ij} + \mu_{ij} m_{ij} + \sum_{i=1}^{I} \sum_{k=1}^{K} \gamma_{ik} e_{ik} + \sigma_{ik} d_{ik} \right\}$$

The problem LP-B has $I \times (J + 2K)$ constraints and $K + I \times (2K + 3J)$ variables. For our example with $I = 6$, $J = 7$ and $K = 5$ this means 102 constraints and 191 variables. The problem LP-P has $I \times (J + K)$ constraints and $K + I \times (2K + 2J)$ variables. For our example this works out to 72 constraints and 149 variables. LP-P is particularly easy to solve since one can always get a reasonable first basic solution by guessing the y_k and then choosing the d_{ik}, e_{ik}, m_{ij}, and n_{ij} to satisfy the constraints.

The duals of both LP-B and LP-P appear to be easier to solve. The dual variables for each problem and each set of constraints is shown in Tables (A-1) and (A-2) respectively. The dual program for LP-B is

$$\text{(A-8)} \quad \text{maximize} \quad \sum_{i=1}^{I} \sum_{k=1}^{K} v_{ik} p_{ik} + \sum_{i=1}^{I} \sum_{j=1}^{J} q_{ij} b_{ij}$$

$$\text{subject to} \quad \sum_{k=1}^{K} u_{ik} f_{kj}^{i} + q_{ij} \leq 0 \qquad \text{for all } (i,j)$$

$$- \sum_{i=1}^{I} u_{ik} w_{ik} + \sum_{i=1}^{I} v_{ik} w_{ik} \leq \lambda c_k \qquad \text{for all } k$$

$$- (1 - \lambda)\gamma_{ik} \leq v_{ik} \leq (1 - \lambda)\sigma_{ik} \qquad \text{for all } (i,k)$$

$$- (1 - \lambda)\pi_{ij} \leq q_{ij} \leq (1 - \lambda)\mu_{ij} \qquad \text{for all } (i,j)$$

Dual of LP-B

The dual of LP-B has $K + I \times J$ constraints, $I \times (2K + J)$ variables, and $I \times (K + J)$ of these variables have upper and lower bounds. In our example this works out to 47 constraints, 102 variables, and 72 variables subject to upper and lower bound constraints.

The dual of LP-P is shown below.

(A-9) maximize $\sum_{i=1}^{I} \sum_{k=1}^{K} v_{ik} p_{ik} + \sum_{i=1}^{I} \sum_{j=1}^{J} q_{ij} b_{ij}$

subject to $\sum_{i=1}^{I} v_{ik} w_{ik} + \sum_{i=1}^{I} \sum_{j=1}^{J} q_{ij} g^{i}_{kj} w_{ik} \leq \lambda c_k$ for all k

$$-(1-\lambda)\gamma_{ik} \leq v_{ik} \leq (1-\lambda)\sigma_{ik} \qquad \text{for all } (k,i)$$

$$-(1-\lambda)\pi_{ij} \leq q_{ij} \leq (1-\lambda)\mu_{ij} \qquad \text{for all } (i,j).$$

Dual of LP-P

This problem has K constraints and I × (J + K) variables with upper and lower bounds. In our example, this would be 5 constraints and 72 variables with upper and lower bounds. This problem may be quite easy to solve.

APPENDIX B

UNCONSTRAINED QUADRATIC MODELS

This appendix outlines two methods for selecting an allocation using the quadratic penalty functions. The two models are similar to the two linear programming models; they use the billet sharing or people sharing rules to go from an aggregate problem to a detailed allocation.

The models do not have any inequality contraints. This is to insure that the solution can be obtained quickly. A full model with inequality constraints might appear to be more appropriate, however, we must recall our ultimate objective is to calculate allocations in a relatively simple way. The model is not built on exact premises and it does not use precise data. It is therefore not terribly important to be exact in choosing an allocation. We hope to have an easy and consistent way of choosing allocations so we can compare the effects of changing assumptions on the allocations.

Both models calculate the billet assignments x_{ij} and the accession rules y_k. The actual allocation is determined by the billet sharing rule in model UQ-B

Rule B.

(B-1) $$a^i_{kj} = f^i_{kj} x_{ij} \ .$$

In model UQ-P, the people sharing rule is used

Rule P.

(B-2) $$a^i_{kj} = g^i_{kj} w_{ik} y_k$$

These rules are discussed in Appendix A.

The objective in these quadratic models is based on symmetric penalties; i.e. $\theta_{ij} = \psi_{ij}$ and $\phi_{ik} = \delta_{ik}$. The objective is a combination of two terms: the cost term and a penalty term

(B-3) $$\lambda \sum_{k=1}^{K} c_k y_k + (1 - \lambda)$$

$$\times \{ \sum_{i=1}^{I} \sum_{j=1}^{J} \mu_{ij} (x_{ij} - b_{ij})^2 + \sum_{i=1}^{I} \sum_{k=1}^{K} \gamma_{ik} (w_{ik} y_k - p_{ik})^2 \}$$

where

(B-4) (i) $$\mu_{ij} = (\theta_{ij} b_{ij})^{-2}$$

(ii) $$\gamma_{ik} = (\phi_{ik} p_{ik})^{-2} .$$

In the UQ-B model we minimize (B-3) subject to the conservation constants.

(B-5) $$\sum_{j=1}^{J} f^i_{kj} x_{ij} - w_{ik} y_k = 0 \quad \text{for each} \quad (i,k).$$

The people sharing model, UQ-P, minimizes (B-3) subject to the constraints

$$x_{ij} - \sum_{k=1}^{K} g_{kj}^{i} w_{ik} y_k = 0 \qquad \text{(B-6)}$$

In UQ-P we can substitute (B-6) directly into (B-3) and thus solve a completely unconstrained problem. In UQ-B we minimize (B-3) subject to (B-5).

APPENDIX C

SAMPLE SOLUTION

This section contains some sample solutions using the unconstrained minimization described in Appendix B.

The data, b_{ij}, f^i_{kj}, w_{ik}, p_{ik}, are described in the paper. The p_{ik} and g^i_k, are calculated in the manner suggested in Section 12.

The solutions are based on data that is largely subjective and does not, in any way, pretend to capture the situation that exists in the Navy. The intent of this section is to demonstrate the feasibility of the scheme.

The target errors are expressed in percentage terms. The 10,000% target error indicates that we do not care very much about meeting billit or personnel targets in the first two stages.

Stage \ Job	1	2	3	4	5	6	7
ENS	10000	10000	10000	10000	10000	10000	10000
LTJG	10000	10000	10000	10000	10000	10000	10000
LT	10	8	6	4	4	5	6
LTCDR	8	6.4	4.8	3.2	3.2	4	4.8
CDR	7	5.6	4.2	2.8	2.8	3.5	4.2
CAPT	6	4.8	3.6	2.4	2.4	3	3.6

Table C-1. The inputs θ_{ij} in %; for billet requirements

Stage \ Job	1	2	3	4	5
ENS	0	10000	10000	10000	10000
LTJG	10000	10000	10000	10000	10000
LT	10	7.5	4.5	5	6
LTCDR	8	6	3.6	4	4.8
CDR	7	5.25	3.15	3.5	4.2
CAPT	6	4.5	2.7	3	3.6

Table C-2/ The inputs ϕ_{ik} in %; for manpower inventories

The billet assignments and the actual percentage errors using the people share rule are

STAGE/JOB	1	2	3	4	5	6	7
ENS	1444	0	2231	1000	1791	972	0
LTJG	1498	0	1989	895	1669	908	1
LT	2164	383	2380	1976	2021	992	399
LTCDR	1242	444	1246	586	977	395	452
CDR	1108	482	1000	489	578	63	884
CAPT	540	529	308	150	0	0	510

Table C-3. The billet assignment, x_{ij}

STAGE/JOB	1	2	3	4	5	6	7
ENS	-253	0	- 6	-45	-86	-87	0
LTJG	- 47	0	- 6	-22	-16	25	20
LT	- 20	- 1	-14	-27	43	29	37
LTCDR	17	26	18	28	45	28	38
CDR	- 7	- 3	- 8	2	15	-15	2
CAPT	9	- 8	4	0	0	0	-32

Table C-4. The percentage error in meeting billet requirements

The accessions to the five classes are:

Class	1	2	3	4	5
	645	1162	524	926	508

Table C-5. Annual rate of accessions, y_k

These accessions produce the actual personnel inventories and the precentage error in meeting inventories are given below.

Class	1	2	3	4	5
ENS	1274	2296	1035	1829	1003
LTJG	1192	2149	969	1712	938
LT	1613	2906	1311	2316	1269
LTCDR	909	1637	738	1304	715
CDR	789	1421	641	1132	621
CAPT	349	629	284	501	275

Table C-6. Personnel inventory by class and stage, z_{ik}.

Class	1	2	3	4	5
ENS	-345	- 6	-45	-86	-87
LTJG	- 67	- 6	-22	16	25
LT	- 28	-14	-27	43	29
LTCDR	13	18	28	45	27
CDR	- 9	- 8	2	15	-15
CAPT	16	4	0	-48	-10

Table C-7. Percentage errors in meeting personnel inventory budgets.

The solution using the billet sharing rule produced similar results. The billet assignments and % error in billet assignments were

STAGE/JOB	1	2	3	4	5	6	7
ENS	1801	0	2074	940	1388	1015	0
LTJG	1685	0	1941	879	1298	949	1
LT	2280	405	2423	1109	1399	1021	499
LTCDR	1284	370	1294	596	676	493	432
CDR	1115	496	1036	483	101	64	1173
CAPT	493	554	291	147	0	0	492

Table C-8

STAGE/JOB	1	2	3	4	5	6	7
ENS	-340	0	1	-37	-44	-95	0
LTJG	-67	0	-3	-20	35	22	0
LT	-26	-7	-17	-31	61	27	21
LTCDR	14	38	12	26	62	10	40
CDR	-8	-5	-12	4	85	-17	-30
CAPT	17	-13	9	2	0	9	-27

Table C-9

The accessions using billet sharing were

Class	1	2	3	4	5
	638	1187	530	748	550

Table C-10

These accession rates give rise to the personnel inventory and percentage error in personnel levels

Class	1	2	3	4	5
ENS	1261	2345	1048	1478	1087
LTJG	1180	2194	981	1383	1017
LT	1596	2968	1326	1871	1376
LTCDR	899	1672	747	1054	775
CDR	780	1451	649	915	673
CAPT	345	642	287	405	298

Table C-11

Class	1	2	3	4	5
ENS	-340	- 9	- 47	- 51	-102
LTJG	- 67	- 8	- 23	33	19
LT	- 26	-17	- 29	54	23
LTCDR	14	10	1	31	- 24
CDR	17	2	- 2	- 20	-20

Table C-12

The analysis of this output should be directed toward constructive changes in the input data. Can we shift some billet assignments from LT to LTJG and LTCDR? Should we tighten up more on the pilot inventory? Should we change the sharing rules? Is it possible to alter the survivor fractions, and thereby improve the solution?

We see that there are a number of potential questions we can answer. The model is flexible and the calculations are rapid. It should be an excellent tool for analyzing manpower policy.

APPENDIX D

ORGANIZATION OF PROGRAMS

This appendix shows how the programs and files for the optimization problem are set up.

FILE STRUCTURE

Component Number	Dimension	Symbol	Description
1	(K,I,J)	G	People sharing array. +/G is an (I,K) element matrix of ones. G is nonnegative: g^i_{kj} is the fraction of type (i,k) people assigned to job(i,j)
2	(I,J)	B	The billet requirements. b_{ij} is the number of type(i,j) jobs to be filled.
3	(I,J)	θ	The range for a unit error in overfilling billets. If $x_{ij} = (1 + 0.01_{ij})b_{ij}$ there is a unit error. $\theta_{ij} > 0$. Expressed as %.
4	(I,K)	ϕ	The range for a unit error in overfilling personnel inventories $z_{ik} = (1\|0.0\|\phi_{ik})p_{ik}$ a unit error is counted. $\phi_{ik} > 0$. Expressed as %.
5	(K,I,J)	F	The job sharing array. F is nonnegative and +/[1]F is an (I,J) matrix of ones: f^i_{kj} is the fraction of type(i,j) jobs to be filled by type(i,k) people.
6	(I,K)	P	The desired manpower inventory: p_{ik} is the number type(i,k) people desired.

FILE STRUCTURE CONT.

Component Number	Dimensional	Symbol	Description
7	(N,K)	α	The survivor fractions for each of the K classes. N is the maximum LOS.
8	I	S	The last year in each stage. Increasing positive numbers with $S_I = N$.
9	(I,K)	W	The element w_{ik} is the number of years a class k input expects to spend in stage i.
10	K	y	The calculated accessions rate; y_k is the number of accessions in manpower class k.
11	(I,K)	z	The calculated inventory of each type of person. There are z_{ik} type (i,k) people.
12	(K,I,J)	A	The calculated allocation: a^i_{kj} is the number of type (i,k) people filling type(i,j) jobs.
13	(I,J)	X	The calculated billets filled. There are x_{ij} people in job(i,j).
14	(K,I,J)	T	The target allocation. We want t^i_{kj} type(i,k) people in type(i,j) jobs. If things are consistent +/T equals P and +/[1]T equals B.
15	(I,J)	ψ	The range for a unit error in under filling billets. If $x_{ij} = (1-\psi_{ij})b_{ij}$ then an error of 1 is counted. $\psi_{ij} \geq 0$.

FILE STRUCTRE CONT.

Component Number	Dimensional	Symbol	Description
16	(I,K)	δ	The range for a unit error in underfilling personnel inventories. If $z_{ik} = (1-\delta_{ik})p_{ik}$ then a unit error is counted. $\delta_{ik} \geq 0$.
17	(K,I,J)	C	The element c^i_{kj} is the total cost of having a type (i,k) person in job(i,j) for one year.

FUNCTIONS

Function	Uses	Computes	Syntax and Description
BILQUAD	B, θ, ϕ, F, P, W	Y, X, Z, A	BILQUAD 'FILENAMS' Calculates "optimal" accessions y, personnel inventory (z), and allocation (A), given the data, objective, and the billet share rule (F).
BILSIM	F,Y,X	Z,A	BILSIM'FILENAMS' Given Y and X (which presumably satisfy the conservation constraint), calculates allocation (A) and personnel inventory (Z), using the billet sharing rule.
PGREC	P, G	B, F, T	PGREC 'FILENAMS' reconciles the file elements B, G, and T with P and G
BFREC	B, F	P, G, T	BFREC'FILENAMS' reconciles the file elements P, G, and T with the file elements B and F in the same file.
PEOQUAD	G, B, θ, ϕ, F, P, W	Y, X, Z,A	PEOQUAD 'FILENAMS' calculates optimal accessions (y), personnel inventory (z), billet staffing (x), and allocation (A), given the data, and use of the people share rule.

FUNCTIONS CONT.

FUNCTION	USES COMPONENTS	CALCULATES COMPONENT	SYNTAX AND DESCRIPTION
PEOSIM	G, Y, W	Z, X, A	PEOSIM'FILENAMS' calculates inventory z, staffing x and allocation A, where accessions and people share rule are given.
WAITS	α, S	W	WAITS 'FILENAMS' Given the survivor fractions (α) and stage definitions (s), calculates w the expected waiting time in each state.
PEOPRONT	P, Z	$100\times(D-z)\div P$	PEOPRONT 'FILENAMS' calculates the percentage error in inventory. Compare with ϕ.
BILPRONT	B, X	$100\times(B-x)\div B$	BILPRONT 'FILENAMS' calculates the percentage error in meeting billet requirements. Compare with θ.
ROUND			ROUND XXX takes any array and rounds elements to integers.

REFERENCES

[1] Grinold, R. C., and K. T. Marshall. Manpower Planning, North-Holland (1977).

[2] Holt, C.C., F. Modigliani, J. F. Muth, and H.A. Simon, Planning Production Inventories and Work Force, Prentice Hall (1960).

[3] Lee, S. M. Goal Programming for Decision Analysis, Auerback (1972).

INITIAL DISTRIBUTION LIST

	No. of Copies
Defense Documentation Center Cameron Station Alexandria, VA 22314	2
Library, Code 0142 Naval Postgraduate School Monterey, Ca. 93940	2
Library, Code 55 Naval Postgraduate School Monterey, Ca. 93940	1
Dean of Research Code 012 Naval Postgraduate School Monterey, CA 93940	1
Director, Total Force Management Control/Analysis Div. (OP-10) Office of DCNO(MPT) Department of the Navy Washington, D.C. 20370	1
Director, Total Force Planning Div. (OP-11) Office of DCNO(MPT) Department of the Navy Washington, D.C. 20370	1
Director, Total Force Program Div. (OP-12) Office of DCNO(MPT) Department of the Navy Washington, D.C. 20370	1
Director, Military Personnel/Training Division (OP-13) Department of the Navy, Office of DCNO(MPT) Washington, D.C. 20370	1
Dr. A. Charnes Dept. of Math. University of Texas Austin, TX 78712	1

	No. of Copies
Prof. M. Cooper Dept. of Ind. Eng. and CR Inst. of Technology Southern Methodist University Dallas, Texas 75275	1
Mrs. Mary Snavely Dixon Office of DASN (Manpower) Department of the Navy Washington, D.C. 20350	1
LCDR Lee Gunn (OP-132C1) Surface Officer Program Manager Office of DCNO(MPT) Arlington Annex Washington, D.C. 20370	2
LCDR Gary Johnson (OP-136C1) Office of DCNO(MPT) Arlington Annex Washington, D.C. 20370	2
Dr. Robert F. Lockman Center for Naval Analyses 1401 Wilson Blvd Arlington, VA 22209	1
Mr. N. K. Malehorn (OP-102) Office of DCNO(MPT) Arlington Annex Washington, D.C. 20370	1
Professor James G. March School of Business Administration Stanford University Stanford, CA 94305	1
Dr. Kneale T. Marshall (OP-01T) Office of DCNO(MPT) Arlington Annex Washington, D.C. 20370	1
CAPT Rodney B. McDaniel (OP-110) Office of DCNO(MPT) Arlington Annex Washington, D.C. 20370	1
R. J. Niehaus 3300 N. Agindon St., Arlington, VA 22207	1

	No. of Copies
LCDR Daniel Parker (OP-130D2) Office of DCNO(MPT) Arlington Annex Washington, D.C. 20370	1
Alfred S. Rhode, Vice President Information Spectrum Inc., 1745 South Jefferson Davis Highway Suite 401 Arlington, VA 22202	1
Dr. Bernard D. Rostker PDASN (M&RA) Department of the Navy Washington, D.C. 20350	1
Joe Silverman (Code 303) Navy Personnel R&D Center San Diego, CA 92152	1
Prof. Robert E. Stanford Dept. of Management Auburn University Auburn, AL 36830	1
Prof. R. Grinold Dept. of Business Administration University of California Berkeley, CA 94720	10
Dr. T. C. Varley (Code 434) Office of Naval Research 800 N. Quincy St. Arlington, VA 22217	1
Naval Postgraduate School Monterey, Ca. 93940	
Attn: J. Arima, Code 54	1
C. Jones, Code 54	1
P. Carrick, Code 54	1
R. Elster, Code 54	1
G. Thomas, Code 54	1
L. Cavaiola, Code 55	1
D. Gaver, Code 55	1
P. Jacobs, Code 55	1
P. Lewis, Code 55	1
P. Milch, Code 55	20
R. Stampfel, Code 55	1
M. Sovereign, Code 55	1